22p.

X63-11413
Code-2d

NASA TT F-8201

"Available to U.S. Government Agencies and U. S. Government Contractors Only."

ON PARTICLE ACCELERATION IN THE EARTH'S OUTER RADIATION BELT

by I. P. Ivanenko and V. P. Shabanskiy

FACILITY FORM 602
N71-71435 (ACCESSION NUMBER)
22 (PAGES)
(NASA CR OR TMX OR AD NUMBER)
(THRU)
None (CODE)
(CATEGORY)

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON
April 1962

"Available to U.S. Government Agencies and U. S. Government Contractors Only"

DRAFT TRANSLATION

NASA TT F-8201

APR 2 1962

N71-71435

ON PARTICLE ACCELERATION IN THE EARTH'S OUTER RADIATION BELT

(Ob uskorenii chastits vo vneshnem radiatsionnom poyase Zemli)

Geomagnetizm i Aeronomiya,
Tom I, No. 6,
pp. 888- 896,
Izad-vo A. N.SSSR, 1961

by I. P. Ivanenko,
V. P. Shabanskiy.

ABSTRACT

Conditions in the outer and inner radiation belts of the Earth are compared. A certain stability of the outer radiation belt is stressed. The acceleration of particles in the outer radiation belt may take place under the effect of hydromagnetic wave, excited at the boundary between the geomagnetic field and the interplanetary plasma. The cause of death of sufficiently fast protons and electrons is found to be the result of their drift toward denser atmosphere layers under the effect of hydromagnetic waves.

COVER-TO-COVER TRANSLATION

The appearance of the inner radiation belt and its existence are sufficiently well explained by the neutron hypothesis [1]. The energetic composition of particles, and the great stability in radiation

intensity of the belt during magnetic storms agree well with the representation within the basic framework of the neutron mechanism. As to the outer radiation belt, much of it still remains vague.

The understanding of processes regulating the intensity, the mass and energy spectrum of particles, and the configuration of the outer radiation belt is complicated by insufficient knowledge of its characteristics through experiments. Without going into the enumeration of its generally well-known properties, we shall underline here a single fact, which in our opinion deserves attention.

The density of the cold plasma (atmosphere) of the inner radiation belt, extending from 10^3 to $6 \cdot 10^3$ km in the equatorial plane, is of the order of $n_c \sim (10^7 \div 10^5)$ cm^{-3}. If we admit the atmosphere temperature being of the of $\sim 10^4$ degrees ($\varepsilon_c \sim 1$ eV), the energy of the cold plasma will be of the order of $E_c = n_c \varepsilon_c \sim (10^{-5} \div 10^7)$ erg/cm^3.

The energy density of the magnetic field is $H^2/8\pi \sim 10^{-4}$ erg/cm^3 (at the equator at a distance $r = 1.8\, r_o$, where r_o is the radius of the Earth, $H \sim 0.05$ gauss).

The density of protons with energy greater than 40 MeV ($n_p > 40$ MeV) $\sim$ $\sim 2 \cdot 10^{-6}$ cm^{-3}. That is why the energy of the "hot" proton plasma component forming the radiation belt is $E_p \sim 3 \cdot 10^{-10}$ erg/cm^3.

It is possible that electrons also exist in the inner belt, but this cannot be stated with total reliability as yet. Their density is determined by the magnitude: $n'_e (> 30$ keV$) \sim 2$ cm^{-3}, $n''_e (> 600$ keV$) \sim 3 \cdot 10^{-3}$ cm^{-3}. The corresponding energy is $E'_e = \varepsilon'_e n'_e \sim 1.6 \cdot 10^{-7}$ erg/cm^2, $E''_e = n''_e \varepsilon''_e \sim$ $\sim 5 \cdot 10^{-9}$ erg/cm^3.

Let us introduce still another characteristic — the Debye radius of the hot component:

$$r_{Dp} = \sqrt{\frac{\varepsilon_p}{4\pi e^2 n_p}} \sim 1{,}5 \cdot 10^9 \text{ см}, \quad r_{De} = \sqrt{\frac{\varepsilon_p}{4\pi e^2 n_e}} \sim 3 \cdot 10^4 \text{ см}.$$

Such determination of the Debye radius of only a single hot component has a specific sense. The Debye radius usually implies the distance over which all plasma charges may be divided owing to the mean thermal motion (The Debye radius of plasma's cold component is $r_D = \sqrt{\varepsilon_e / 4\pi e^2 n_e} \sim 0{,}2$ см). In the current case the Debye radius determines the distance at which only one-sign particles of the plasma's hot component may for instance drift away from the examined plasma volume.

Thus, the energy of the hot component (fast protons) is by $3 \div 5$ orders lower than that of the cold component, which is in its turn by $1 \div 3$ orders lower than the magnetic field energy. The Debye radius of the hot component ($r_{Dp} = 1.5 \cdot 10^9$ cm) is larger than the dimensions of the region occupied by the inner radiation zone. That is why the hot component does not really determine the plasma, in a proper sense of the word. It is well known that the magnetic trap holds such particles quite well, without being wrecked. Thus the questions of plasma instability on account of energy component do not even arise for the inner belt. The motion of the belt's lines of force is being controlled by plasma's cold component, much less subject to fluctuations on account of exterior factors. This explains in particular the great stability of the inner radiation belt. The motion of energy component in the magnetic trap may be viewed as a flow of separate particles,

without accounting their electric interacting, leading to neutrality. The cold plasma serves only as background for these particles' flow, determining the friction force and lifetime of high-energy protons and electrons.

In the outer radiation belt, extending from three to seven Earth radii, the conditions are different. There, the density of particles and the energy of the cold component are respectively equal: $n_c \sim 10^2 \div 10^3$ cm^{-3}, $\varepsilon_c \sim 1$ eV, $E_c = n_c \varepsilon_c \sim 1.5\ (10^{-9} \div 10^{10})$ erg/cm^3.

The energy density of the magnetic field $H^2/8\pi \sim 10^{-6}$ erg/cm^3 (at $H_o \sim 0.4$ gauss at the equator). At the same time $H = H_o (r_o/r)^3 \sim$ $\sim 6 \cdot 10^{-3}$ gauss at a distance $r = 4r_o$).

As to the hot component, it is more or less well known that the basic part of energy is contained in electrons, but there are divergences in the questions about the spectrum and the mean number of energetic electrons, which are explained by the different interpretations of experimental data. Thus, according to Van Allen [2], the number of electrons with energies exceeding 30 keV n'_e ($>$ 30 keV) ~ 10 cm^{-3} ($E'_e = n'_e \varepsilon'_e \sim 8 \cdot 10^{-7}$ erg/cm^3), while n''_e ($>$ 200 keV) $\sim 3 \cdot 10^{-3}$ cm^{-3} ($E''_e \sim 10^{-8}$ erg/cm^3). The first figure was subject to doubt. Another interpretation [3] gave n_e ($>$ 50 keV) $\sim 2 \cdot 10^{-3}$ cm^{-3} ($E_e = n_e \varepsilon_e \sim 1.6 \cdot 10^{-10}$ erg/cm^3), i.e. a value lower by almost four orders. Currently, Soviet investigators reach the same value.

The Debye radius of these electrons respectively is $r'_D \sim$ $\sim 5 \cdot 10^4$ см, $r''_D \sim 5 \cdot 10^6$ см and $r_D \sim 6 \cdot 10^6$ см. The estimate of the possible number of protons in the outer belt is

$$n_p(>60\ MeV) \lesssim 10^{-8}\ см^{-3}\ (E_p = n_p \varepsilon_p \sim 10^{-12}\ эрг/см^3).$$

It may be seen from these data, that the energy density of electrons with energy of several tens of keV is either by one order (according to more recent estimates), or by three orders (according to Van Allen) higher than the energy of the cold component, while the energy density of electrons with $\varepsilon > 200$ keV is by 1 ÷ 2 orders higher than the energy of the cold component. Besides, the Debye radius of the energy component is significantly smaller than the dimensions of the outer zone at either estimate ($L \sim 3 \cdot 10^9$ cm). That is why the energy component represents a plasma with all its characteristic properties, so that during the solution of questions related to possible particle acceleration, to their injections from solar fluxes, to instability or unsteadiness of this region, etc., it is not possible to start from the representation of a separate particle flow in the geomagnetic field.

Relative Stability of the Inner Radiation Belt

The motion of the magnetic field's lines of force in the outer radiation belt is controlled by the plasma's hot component. The field's configuration relative to solar corpuscular streams, flowing about the geomagnetic field is convectively stable, provided we apply here the criterion of boundary stability between the plasma and the magnetic field [4] (the magnetic lines of force are convex toward the solar plasma). The solar corpuscular stream with a density $n \sim 10^2 \div 10^3$ cm^{-3} and a velocity $v_0 \sim 10^8$ cm/sec, presses out the magnetic field to approximately the distance from the center of the Earth, determined by

the equality $H^2/8\pi \sim 2m_p v_o^2 n$, whence $r \sim (6 \div 10) r_o$. At the same time the following picture of phenomena, taking place in the near-terrestial space, can be visualized.

The pressing out of the geomagnetic field by the solar corpuscular stream is translated by the increase at the Earth of the magnetic field's horizontal component, which is being observed during the first phase of a magnetic storm with a sudden commencement. After the departure of the solar stream from the Earth's environment the geomagnetic lines of force are widened by the plasma of the outer radiation belt within which the particles' energy density increased during the storm. Had a sharp boundary existed between the outer belt's plasma and the interplanetary space, such field configuration would be convectively unstable in the absence of stabilizing action of the ionosphere (which is to be discussed below), account being taken that the magnetic lines of force are convex toward the side of the plasma). In fact, there is no sharp boundary. In that case, the trap with closed lines of force will be convectively stable in the case [5], when

$$dP/dV < \frac{\gamma P}{|V|} \qquad V = -\oint \frac{dl}{H}, \tag{2,1}$$

where P is the pressure of the plasma; $\gamma = 5/3$ is the adiabat index; V is the integral along the closed line of force. This expression is valid for the case of weak pressure, when $H^2/8\pi P \gg 1$, which is fulfilled for the internal regions of the outer radiation belt even when the plasma energy is overrated ($E_e' \sim 8 \cdot 10^{-7} erg/cm^3$).

For a point dipole $V \sim r^4$, and from (2.1) we have

...

$$-\frac{d\ln P}{d\ln r}<4\gamma \qquad P<\frac{C}{r^{4\gamma}}\approx\frac{C}{r^{7}}, \tag{2,2}$$

where C is a constant. Therefore, if pressure drops slower than approximately the seventh power of the distance, the trap will be stable for the outer zone. One might think that upon solar plasma departure, this condition would be deranged in the outermost parts of the zone, which ought to lead to a mutual displacement of the field's tubes of force, and to partial exit of plasma from the trap.

However, this is impeded by the stabilizing action of the regions through which pass the lines of force at low altitudes. If in some region the field's lines of force pass through a solid body with infinite conductivity, the condition of convective stability takes the form [5]:

$$-\frac{\nabla P\nabla V}{V}<\gamma P\left(\frac{\nabla V}{V}\right)^2+\frac{\pi H^2}{4L}, \tag{2,3}$$

where L is the trap's characteristic dimension.

For a dipole, this gives

$$-\frac{d\ln P}{d\ln r}<4\gamma+a\frac{H^2}{8\pi P} \qquad (a\sim 1). \tag{2,4}$$

The regions in which $H^2/8\pi P \gg 1$ are known to be stable. However in the outer region of the radiation belt, the relation $H^2/8\pi P$ may be close to the unity. Besides, in a real case the geomagnetic lines are not "frozen" in the "solid body". The tying to the terrestrial nucleus of the outer field's lines of force is absent on account of the existence of a neutral interlayer between the Earth and the ionosphere. Therefore, only the ionosphere may exert a stabilizing action. But the energy of the radiation belt's particles contained in the neighboring tubes of force

which narrow down considerably toward the ionosphere, may be sufficient for a slow displacement of the mass of the ionosphere at these tubes' convection. This may be one of the causes of occurrence of ionospheric winds.

From the aforesaid it follows, that one more channel of particle escape from the geomagnetic trap may apparently exist side by side with the ionizing losses and with particle exit through magnetic plugs, on account of relative convective instability of the system, and of tube of force egress outside with the plasma's energetic component.

Particle Acceleration in the Outer Radiation Belt

There is one question though, to which no well-defined satisfactory answer has been found so far, and that is the origin of the energy component of the outer radiation belt. The hypothesis of energy component appearance of the outer radiation belt at β-decay of albedo neutrons of cosmic rays [3] is apparently inconsistent, firstly, because of insufficient density of the neutron flux to assure a stationary intensity of the outer radiation belt, when the electron lifetime in them is known, secondly — on account of the non-conformity between the β-rays' energy spectrum and the electron spectrum of the belt.

There is no doubt that the emergence of the energy component is linked, if only partly, with sunspots interacting with the Earth's magnetic field. The increase in radiation intensity within the outer belt, and its shift in space correlates well with geomagnetic storms [6]. The fact that the original radiation level settles after the storm [7] may be understood

from that viewpoint, if we admit the existence of a permanent solar wind of lesser intensity, and not only at times of solar flares. Moreover, the Earth's translation relative to the interplanetary gas may serve as a perturbing source of the field, leading to high-energy particle emergence. This source is quite acceptable from the energetic viewpoint. The total energy of electrons in the zone with a volume $V \sim 5 \cdot 10^{28}$ cm^3 (a sphere with a radius $r = 4r_o$) is $\varepsilon_e n_e V \sim 5 \cdot 10^{18}$erg. If we take for electron lifetime $t_e^{(n)} \sim 7 \cdot 10^6$ sec (ionization losses of an electron with 10 keV energy in a gas with a density $n \sim 10^3 cm^{-3}$), the energy lost in a second by particles of the belt is

$$_e nV / t_e^{(n)} \sim 10^{12} \text{erg/sec}.$$

The energy of the solar flux, reflected from the surface S of a circle with a $10r_o$ radius, is

$$2 m_p v_o^2 n_o s v_o \sim 10^{20} \text{ erg/sec}.$$

for a particle density in the flux $n_o \sim 1$ cm^{-3}, a flux velocity $v_o \sim 10^8$cm/sec $s \sim 10^{20}$cm^2. If in place of solar flux velocity we substitute the value $v_o \sim 3 \cdot 10^6$ cm/sec (velocity of the Earth relative to the interplanetary gas), we shall have

$$2 m_p v_o^2 n_o s v_o \sim 10^{16} \text{erg/sec}.$$

Hence we may see, that in both cases there is enough of that energy with an excess, so that in converting into particle energy of the radiation belt, it could make up the leakage.

One may figure two alternative ways of outer radiation belt emergence under the effect of solar plasma flux, flowing about the geomagnetic field. The first consists in the possibility of energetic particle injection from the solar plasma into the geomagnetic field. At that time, a simple diffusion of solar plasma in the geomagnetic field cannot be achieved. At $n \sim 10^3$ cm^{-3} and $T \sim 10^4$ deg. the time of electron relaxation of solar plasma is $\tau_{el} \sim 10^3$ sec, the conductivity is $\sigma \sim e^2 n \tau / m_e \sim 10^{14}$ CGSE and the time of plasma diffusion in the field at a depth $x \sim 10^9$ cm is of the order of $t = 4\pi\sigma x^2/c \sim 10^{12}$ sec. This time is by several orders greater that the accretion time of radiation belt's intensity during a storm. That is why the injection mechanism must be different. There forms between the plasma and the pressed out geomagnetic field a transition layer [8], only $x \sim c/\omega_0 = \sqrt{m_e c^2/4\pi e^2 n_0} \sim 10^5$ см, thick, in which a temporary energy transfer from protons to electrons takes place.

Since the energy of the flux' protons $m_p v_0^2/2$ is of the order of several tens of keV, the electron energy, observed in the outer radiation belt is of the same order. This led to the idea, more than once evoked by various authors, of the possibility of similar origin for energetic electrons in the outer belt. It was attempted to remove the difficulty appearing on account of the small width of the transition layer and its convective stability by bringing up the beam instability mechanisms. Similar processes may apparently be admitted to the outermost parts of the radiation belt, which is sometimes called the third radiation belt. But it is difficult to think that such mechanism might engulf the deeper layers of the belt, lying at distances of 3—5 Earth's radii in the geomagnetic equator plane.

Besides, it must be noted that the beam instability may apparently only lead to a widening of the transition zone (by a shorter time than the diffusion). But at the same time, the mechanism of energy transfer from protons to electrons ceases to act: the wider the layer, the less energy transfer takes place, and the recourse to such injection mechanism loses its sense.

Another idea in the same context consists in that separate plasma clusters, endowed with a great kinetic energy [9], may intrude the geomagnetic field. However, even this mechanism is unsatisfactory because the probability of emergence of such fluctuating clusters with a density substantially exceeding the mean density of the flux, is small.

Therefore, neither the meachanism of electron heating by way of transfering them the energy from the directed flow of solar flux' ions, nor the direct injection of solar plasma into the geomagnetic field, may be realized under real conditions.

The second means consists in that hydromagnetic waves may emerge in the region occupied by the field, propagating toward the Earth, as a result of solar flux interaction with the geomagnetic field in that region. They may lead to particle acceleration and to the formation of the outer radiation belt.

Thus the question is whether or not energy dissipation takes place in the waves, excited in the near-terrestrial plasma, i.e. can there be a shock wave, with a front narrower than the region's characteristic dimension. Thus, even the cold component

The relaxation time for protons of plasma's cold component $t_{cp} \sim$ $\sim 10T^{3/2}/n \ln \Lambda \sim 10^3$ sec at $T \sim 10^4$ deg, $n \sim 10^3$, and the Coulomb

logarithm $\ln \Lambda \sim 10$. This means that the length of the path is $\lambda_{cp} \sim v_{cp} t_{cp} \sim 2 \cdot 10^9$ см, thus of the order of the dimensions of the region itself. For the energy component, the length of the path is substantially greater than the region's dimensions. Thus, even the cold component forms a collisionless plasma.

However, there are experimental indications that the width of the shock wave in a plasma with a frozen-in magnetic field, propagating in a direction perpendicular to the field, is substantially less than the length of the free path, and near the protons' Larmor radius.

On the other hand, it has been shown in reference [10] that the width of the shock wave's front, at which energy dissipation takes place, is of the order

$$l \sim \frac{c}{\omega_0} \left(\frac{H^2}{8\pi P} \right)^{1/2} \qquad (3,1)$$

for waves with Mach number

$$M^* > 1 + (8\pi P/H^2)^{1/2}, \qquad (3,2)$$

where c is the speed of light, $\omega_0 = \sqrt{4\pi e^2 n/m_e}$ is the Debye frequency. For the cold component

$$(n \sim 10^3)\ \omega_{0c} \sim 2 \cdot 10^6\ sec^{-1},\ H^2/8\pi P \sim 7 \cdot 10^2 \qquad l_c \sim 4 \cdot 10^5\ см,$$

for the energy component

$$(n_e \sim 10^{-3}) \quad \omega_0 \sim 2 \cdot 10^3\ sec^{-1} \quad and \quad l \sim 4 \cdot 10^8\ см.$$

Hence it may be seen, that even at a least favorable estimate of electron density, the expected width of the shock wave's front, at which energy dissipation takes place, is less than the dimension of the examined region even for the energy component.

The question of particle acceleration in a collisionless plasma

by shock waves in a magnetic field was examined in reference [11], where the problem of plasma heating by pistons, oscillating in a direction perpendicular to that of the nagnetic field, is studied. The energy accretion is obtained as a result of the fact, that compression takes place along a shock adiabate, while expansion by the Poissons adiabate runs more slantly. We consider it necessary to note here, that aside from numerous misprints, allowed to remain in that paper, a substantial error has stolen in formula (3.5). In fact, the magnitude δ_2 of the relative pressure variation after a full compression and expansion cycle, equals (with a precision to a factor of the order of the unity) δ_1^3 and not δ_2^2, where δ_1 is the relative pressure increase at the front of the weak shock wave. This corresponds to the fact, that irreversible processes, linked with entropy rise at the weak shock wave's front, is a small magnitude of the third order in comparison with the magnitude of pressure jump. We then shall have with a precision to coefficient of the 1 order

$$\Delta\varepsilon/\varepsilon = \Delta P/P = \delta_2 \sim \delta_1^3 \sim (u/s)^3 \sim (u^2/\varepsilon)\ (u/s),$$

where $\Delta\varepsilon$ is the energy accretion, u the piston's velocity, s - the wave's velocity. For one piston's oscillation we have for the gas mass

$$m\,\Delta\varepsilon \sim mu^2(u/s).$$

Thus, formulas in reference [11], where u^2 enters, must be corrected by the factor u/s. The energy accretion for one oscillation of the piston and 1 g. of gas, is

$$\Delta\varepsilon = \frac{\gamma+1}{12}\frac{u^3}{s^*}, \qquad (3,3)$$

where γ is the adiabatic index, and u — the mean piston's velocity, assumed

to be much lower than that of the shock wave, i.e. the rate of hydromagnetic disturbance propagation for a weak shock wave is

$$s^* = \sqrt{s^2 + s_a^2} = \sqrt{\gamma RT/\mu + H^2/4\pi m_p n}$$

Let us examine three cases.

1. The frequency of piston's oscillations $\nu \ll s^*/L$, where L is the region's characteristic dimension. In this case the rate of the gas mass m heating is

$$\frac{d\varepsilon}{dt} = \frac{\gamma+1}{12}\frac{mu^3}{s^*}\frac{s^*}{L}. \qquad (3,4)$$

2. Let us assume $\nu \gtrsim s^*/L$. In this case, during the time the front of the first wave did not reach the second piston yet, the first piston will generate a second wave, etc. The heating will be $\nu/(s^*/L)$ times greater (measured in terms of wave "teams" piling on L).

Consequently, in that case

$$\frac{d\varepsilon}{dt} = \frac{\gamma+1}{12}\frac{mu^3}{s^*}\nu. \qquad (3,5)$$

3. The latter formula is only valid for $\nu \ll \nu_{ст}$, where $\nu_{ст}$ is the frequency of collisions between the plasma particles leading to energy redistribution. For a collisionless plasma, the magnitude s^*/l will the role of this characteristic, l being determined according to formula (3.1). It is obvious that at $\nu \gg s^*/l$ the increase in the oscillation frequeny will not lead to heating increase, for it will be limited by the width of the wave's l front, at which dissipation takes place. Thus, in case $\nu \gg s^*/l$.

$$\frac{d\varepsilon}{dt} = \frac{\gamma+1}{12}\frac{mu^4}{s^*}\frac{s^*}{l}. \qquad (3,6)$$

At the same time, the condition (3.2) must be fulfilled, i.e. the wave must not be too weak. Let us now examine to what this condition amounts, and let us also express u through the amplitude of magnetic field oscillations at the condition $H^2/8\pi P \gg 1$, which is fulfilled for the outer zone.

Assuming that the plasma has two degrees of freeedom, i.e. $\gamma = 2$ (as we did earlier for facilitating the computations of reference [11]), we bring together the equations at the front of the hydromagnetic perpendicular wave with the equations, precisely coinciding with those for the shock wave without magnetic field, in which instead of P, E, s and etc. the following magnitudes will enter

$$P^* = P + H^2/8\pi, \quad \varepsilon^* = P^*V/(\gamma-1), \quad s^* = \sqrt{s^2 + s_a^2} = \sqrt{\gamma(\gamma-1)\varepsilon^*} \quad etc...$$

The equation of state ($\gamma = 2$) will then coincide with the equations of state for an ideal gas

$$P^*V^\gamma = PV^\gamma + H^2V^\gamma/8\pi = \text{const},$$

since, because of the freeze-in condition $H^2V^2 = \text{const}$, for $\gamma = 2$.

The correlations at the shock wave front have the form:

$$\frac{P_2^*}{P_1^*} = \frac{2\gamma(M_1^*)^2 - (\gamma-1)}{\gamma+1} \qquad (3,7)$$

$$\frac{v_2}{v_1} = \frac{(\gamma-1)(M_1^*)^2 + 2}{(\gamma+1)(M_1^*)^2}, \qquad (3,8)$$

where v_1, v_2 respectively are velocities of the front relatively to the

gas ahead and behind the front (medium 2), $M_1^* = v_1/s^* = 1 + \delta$ are the Mach numbers. The condition for the weak shock wave $\delta \ll 1$, gives

$$M_1 \ll \sqrt{3}\sqrt{1 + H_1^2/8\pi P_1} \sim \sqrt{3}\,(H/8\pi P)^{1/2},$$

where $M_1 = v_1/s$ — is the standard Mach number. (For the case of plasma in the belt $M_1 \gg 1$).

Expanding (3.8) in series by powers δ, we obtain in the first order: $u/v_1 \approx u/s^* \approx 2\delta/(\gamma + 1)$. For the case $H^2/8\pi P \gg 1$, we have from equation (3.7):

$$\Delta H/H = (H_2 - H_1)/H_1 \approx 2\gamma\delta/(\gamma + 1).$$

Therefore

$$\frac{\Delta H}{H} = \frac{2\gamma\delta}{\gamma+1} = \gamma\frac{u}{s^*}. \qquad (3,9)$$

The condition (3.2) is brought to $\delta \gg (8\pi P/H^2)^{1/4}$ or

$$\frac{\Delta H}{H} > \frac{2\gamma}{\gamma+1}\left(\frac{8\pi P}{H^2}\right)^{1/4}. \qquad (3,10)$$

For the outer belt $\Delta H/H > 10^{-1}$. Formula (3.1) for the width of the front, at which energy dissipation takes place, is only valid for such waves. At lower amplitudes the heating will be less effective.

Formulas (3.4) — (3.6) may be united in a single interpolation formula for the three regions of frequency ν. Expressing in them u according to formula (3.9) by $\Delta H/H$, we finally have:

$$\frac{d\varepsilon}{dt} = \lambda\varepsilon = \frac{\varepsilon}{l^{(H)}}, \qquad (3,11)$$

$$\lambda = \frac{\gamma+1}{12\gamma^3}\frac{m(s^*)^2}{\varepsilon}\left(\frac{\Delta H}{H}\right)^3 \frac{\frac{s^*}{l}\left(\nu + \frac{s^*}{L}\right)}{\frac{s^*}{l} + \nu}, \qquad (3,12)$$

where

...

$$l = \frac{c}{\omega_0}\left(\frac{H^2}{8\pi P}\right)^{1/2}, \quad \omega^2 = \frac{4\pi e^2 n}{m_e};$$
$$s^* = \sqrt{s^2 + s_a^2} = \sqrt{\frac{\gamma RT}{\mu} + \frac{H^2}{8\pi m_p n_0}}.$$

At the same time the condition $l/L \ll 1$ and (3, 10) must be fulfilled.

The rate of energy assembly according to formula (3, 11) must be compared with the rate of losses. Considering that the particle outflow basically takes place on account of ionization losses, we shall have for electrons and protons respectively

$$\frac{d\varepsilon_e}{dt} \sim \frac{\varepsilon_e}{t_e^{(n)}} \sim \frac{4\pi e^4 n \ln\lambda}{m_e v_e}, \qquad (3,13)$$
$$\frac{d\varepsilon_p}{dt} \sim \frac{\varepsilon_p}{t_p^{(n)}} \sim \frac{2\pi e^4 n \ln\lambda}{m_e v_e}, \qquad (3,14)$$

where $\ln\lambda \sim 20$ is the Coulomb logarithm.

If we compute the characteristic times of energy collecting according to formulas (3, 11) — (3.12) for the case $v \ll s^*/L$ and lifetime of protons and electrons t^n_e, t^n_p, we shall obtain the following Table, easy for the comparison.

ε	$t^{(н)}_{e,}$ сек sec	$t^{(н)}_{p,}$ сек sec	$t^{(п)}_{e,}$ сек sec	$t^{(п)}_{p,}$ сек sec
10 кэв (KeV)	$2\cdot10^6$	10^3	$7\cdot10^6$	$1{,}6\cdot10^5$
100 кэв (KeV)	$2\cdot10^7$	10^4	$2\cdot10^8$	$5\cdot10^6$
1 Мэв (MeV)	$2\cdot10^8$	10^5	$6\cdot10^9$	$1{,}5\cdot10^8$
10 Мэв MeV	$2\cdot10^9$	10^6	$1{,}8\cdot10^{11}$	$4{,}5\cdot10^9$
100 Мэв MeV	$2\cdot10^{10}$	10^7	—	—

At the same time we admitted $\Delta H/H \sim 1$ and $s \sim s_a \sim 3\cdot10^7$ cm/sec. Such value of the Alfvén velocity is obtained for $n \sim 10^3$ cm^{-3}, at a $r \sim 4r_0$ distance from the center of the Earth.

It may be seen from the Table, that the time of losses and assmbly of electrons' energy is of the order of 1 keV. The critical velocity of electrons is v_{e0}, above which their acceleration is possible,

is obtained from (3.11) and (3.13):

$$v_{e0} \sim \frac{2\pi\gamma e^4 nL \ln\lambda}{m_e^2 (s^*)^3 \left(\frac{\Delta H}{H}\right)^2}. \qquad (3,15)$$

It corresponds exactly to 1 keV electron energy for the above admitted values. Therefore, the problem of electron heating to that critical energy remains open. There are indications, that for oblique shock waves the mechanism of energy transfer from protons to electrons, which is similar to the transfer mechanism in the layer between the plasma and the field, is partly realized. If the preliminary acceleration really had such an assumed selectivity, this would compensate the too swift energy assembly, by protons, for which the heating mechanism by shock waves is more effective than for electrons.

Let us note still another imprtant circumstance: Shock waves heat energetic particles much more effectively than the thermal energy particles [11, 12]. This explains the belt's plasma subdivision into hot and cold components. The relative pulse accretion of the energetic particle is greater that the relative accretion of the mean pulse of thermal particles, proportional to pressure jump, by about $2/(\gamma - 1)$ times for weak shock waves. If the discharge wave has no steep drop at a distance equal to particle's Larmor radius, the cooling of both the energetic and thermal particles, will be adiabatic. And since the energy accretion is proportional to the third power of the pulse's jump, the increase of energetic particles' pulse will be $8(\gamma - 1)^3$ times greater than that of thermal particles.

As follows from the examined mechanism of acceleration, particles should be accelerating boundlessly, for the depth, to which they outflow in the atmosphere and perish, determined by the reflection point in the magnetic field, depends only upon the angle between particle's velocity and the field, and not on particles' energy. Thus, there should exist some mechanism, limiting the acceleration process from above:

1) Scattering on Alfvén waves, at which the angle between the particle's velocity and the line of force varies;

2) Particle outflow from the magnetic field at tube of force ionization, to which it was referred earlier;

3) Drift of energetic particles toward denser atmosphere layers under the effect of shock waves.

The latter mechanism may constitute a special interest. Let us briefly formulate its substance.

As already noted, it follows from the results of work [12], that when an energetic particle passes the front of a perpendicular shock wave in a plasma with a magnetic field, the particle accelerates significantly more, than thermal particles. This happens in the case, when the particle's Larmor radius is significantly greater than the width of the front. The particle results tied up to the front during a certain time, and is displaced with it, and at the same time it drifts along the front's surface with a velocity of the order of that of the particle itself. Therefore, particles, whose Larmor radius is greater than the front's width, will be driven by waves, moving along the longitude from the magnetic field's periphery, and — what is most important — along the radius in the direction toward

the center of the Earth, into denser atmosphere layers.

The critical value of the pulse p_o of particles, whose Larmor radius is comparable with the width l_c of the front, is determined by the equality $cp_o/eH = l_c$.

At $l_c \sim 4 \cdot 10^5$ cm (for the cold component at $r \sim 4r_o$), this corresponds to protons with subrelativistic energies $\varepsilon_{po} \sim 2 \cdot 10^4$ eV and to relativistic electrons with energy $\varepsilon_{oc} \sim 6$ MeV.

Particles of greater energies must be driven by hydromagnetic waves into denser layers of the atmosphere. This apparently corresponds to the experimental data, obtained on the second cosmic rocket [13], from which it follows, that protons with energies above 30 MeV, and electrons with energies greater than 5 MeV are absent in the outer radiation belt.

****** THE END ******

Translated by ANDRE L. BRICHANT

for the

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

REFERENCES

1. S. N. VERNOV, A. E. CHUDAKOV. UFN, 70, No. 4, 585, 1960.
2. J. A. VAN ALLEN, J. H. LUDWIG & al, Jet Propulsion I, 28, 1958.

   J. A. VAN ALLEN, C. E. McILWAIN, J. H. LUDWIG.. J. Geophys. Res. 64, No. 3, 271, 1959.
3. W. H. HESS. J. Geophys. Res., 65, No. 10, 3107, 1960.
4. L. LONGMAIRE, M. ROSENBLUTH, Ann. Phys., 1, 120, 1957
5. B. B. KADOMTSEV. Geomagn. ustoychivost' plazmy, Preprint IAE, 1960
6. P. ROTHWELL, C. E. McILWAIN. J. Geophys. Res, 65, 3, 799, 1960.
7. R. L. ARNOLDI, R. A. HOFFMAN, J. R. WINCKLER, Ibid., 65, 5, 1960
8. V. P. SHABANSKIY. ZhETF, 40, No. 4, 1058, 1961.
9. S. B. PIKEL'NER., Astronom. Zhurnal, 36, 6, 1134, 1959.
10. A. VEDENOV, E. P. VELIKHOV, R. Z. ZAGDEYEV, Yadernyy sintez, 1, No. 1, 83, 1961
11. V. P. SHABANSKIY. Geomagnetizm i Aeronomiya, 1, No. 4, 483, 1961
12. V. P. SHABANSKIY. Zh.ETF., 41, No. 4(10), 1107, 1961
13. S. N. VERNOV & al., Dokl. A. N.SSSR., 130, No. 3, 517, 1960.

Moscow State University
in the name of M. V. Lomonosov
Institute of Nuclear Physics.

Entered on 19 October 1961.